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1980 J. Phys. A: Math. Gen. 13 713

(http://iopscience.iop.org/0305-4470/13/2/033)

J. Phys. A: Math. Gen., 13 (1980) 713–724. Printed in Great Britain

On an OSp(1, 4) renormalisable theory of supergravity with higher derivatives

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Received 16 January 1979, in final form 25 April 1979

Abstract. We present a supergravity Lagrangian invariant under local orthosymplectic and general coordinate transformations. It is conjectured that the Lagrangian describes a renormalisable theory. New features in this formulation are the introduction of a group-invariant metric tensor and the corresponding connection. As in other higher-derivative theories metric ghosts are present.

1. Introduction

The formulation and development of supergravity theories in recent years has proceeded mainly along two distinct channels. The first is the geometrised, superspace approach (Brink *et al* 1978, Wess and Zumino 1978, Akulov *et al* 1975, Arnowitt and Nath 1978). The second is Poincaré supergravity (Freedman *et al* 1976, Deser and Zumino 1976), in which the Einstein action was coupled minimally to a spin-$\frac{3}{2}$ Rarita–Schwinger field and the field transformations that leave the supergravity action invariant were obtained iteratively. Until recently (Stelle and West 1978, Ferrara and van Nieuwenhuizen 1978), the supersymmetric gauge algebra of Poincaré supergravity could be made to close only on-shell, a defect which led to the invention of the graded algebraic approach (Chamseddine and West 1978, MacDowell and Mansouri 1977). In this framework, which is a direct application of the gauge method to supersymmetry, the underlying group-theoretic structure is made manifest throughout.

The SL(2, $\mathbb{C}$) internal symmetry of Weyl (which on gauging gives rise to a spin-2 gauge field) is replaced by the orthosymplectic symmetry OSp(1, 4). Gauging this supergroup leads to a theory of coupled spin-2 and spin-$\frac{3}{2}$ fields. As in the case of the Weyl SL(2, $\mathbb{C}$) symmetry, general covariance is not mandatory but may be imposed as a further symmetry of the theory, distinct from the invariance under graded OSp(1, 4). Constraints and a Wigner–Inönu contraction are then required to reproduce the action of Poincaré supergravity.

The maximal Lie subgroup of the supergroup OSp(1, 4) is Sp(4), the de Sitter covering group. Further, the homogeneous space OSp(1, 4)/Sp(4) has four anti-commuting elements, parametrised by the four components of the Majorana spinor θ_α. Since coordinate space X^μ in a de Sitter universe can be associated with Sp(4)/SL(2, $\mathbb{C}$), the superspace (X^μ, θ_α) is simply the homogeneous space OSp(1, 4)/SL(2, $\mathbb{C}$). This

† Supported in part by an Italian Government Fellowship.

constitutes the link between the graded algebraic and superspace approaches, and provides part of the motivation for a gauged OSp(1, 4) theory of supergravity.

A theory of massive supergravity based on a gauged orthosymplectic symmetry has been constructed recently (Chamseddine 1977, 1978, Chamseddine *et al* 1978, Gürsey and Marchildon 1978). In this formulation, the SL(2, ℂ) gauge invariance of Einstein gravity is extended to a local OSp(1, 4) symmetry. The Einstein group of general coordinate transformations GL (4, ℝ), or more precisely (since this itself is broken) its O(3, 1) subgroup, is left intact. Then, by spontaneously breaking OSp(1, 4) down to SL(2, ℂ) and demanding that the vacuum be Poincaré-invariant†, a mass for the spin-$\frac{3}{2}$ gauge field is generated‡ via the 'super-Higgs' effect (Chamseddine 1977, 1978, Cremmer *et al* 1978).

Two multiplets of fields are required. The gauge potentials transform under a fourteen-dimensional tensor representation of OSp(1, 4), and a 'super-Higgs' multiplet, necessary to induce spontaneous symmetry-breaking, belongs to a ten-dimensional representation. To restrict the final particle spectrum, non-linear realisations and covariant constraints are used to implement the symmetry breakdown, and to suppress the dynamical role of some of the fields.

In this paper§, we construct a renormalisable, parity-conserving theory of supergravity which is invariant under local OSp(1, 4) and general coordinate transformations. This essentially constitutes the supersymmetric extension of R^2 and $R^{\mu\nu}R_{\mu\nu}$ gravity (Stelle 1977)|| within the orthosymplectic framework. Although taking the vierbein and the Rarita–Schwinger field as the basic dynamical objects, it is nevertheless convenient to define a metric tensor which is invariant under the group action¶.

Higher derivatives are included in the action, leading to propagators with a momentum–space high-energy behaviour of k^{-4}. The one-loop divergences are thereby sufficiently softened for the theory to be renormalisable in the conventional sense. To this end, we find it necessary to define a generalised (Cartan) covariant derivative of the OSp(1, 4) field strengths and to introduce a corresponding connection to preserve general coordinate invariance.

The connection can be solved for in terms of the dynamical fields $L_\mu{}^a$ and ψ_μ. This is done by using the standard constraint that the covariant derivative of the metric vanishes.

One novel feature is that the higher derivatives injected into the theory cause the SL(2, ℂ) gauge field $B_{\mu ab}$ to propagate, unlike the situation in ordinary supergravity where it can be eliminated by an algebraic constraint equation.

The theory considered here has in common with conformal supergravities the pathology of metric ghost states occurring in the particle spectrum, a point to which we shall briefly return in the conclusion. Apart from the question of ghosts, another interpretative difficulty arises from the propagation of the SL(2, ℂ) gauge field. This evidently introduces further spin-2 components into the theory. It is conceivable that

† In actuality, the vacuum transforms under the O(3, 1) subgroup of O(3, 1) × SL(2, ℂ), the O(3, 1) in the direct product coming from the broken group of general coordinate transformations.

‡ A cosmological term is also generated, which must be cancelled if the background is to be Minkowskian.

§ We adopt the conventions of Chamseddine (1977, 1978).

|| However, unlike R^2-gravity, the present theory, as will be seen, contains a propagating SL(2, ℂ) connection. Thus, too direct a comparison with the former may be misleading and is not intended.

¶ This is a departure from the purely geometric (affine) theory in which the metric does not explicitly appear except in the curvature tensor.

this latter problem may be removed by imposing a constraint on the OSp(1, 4) curvature (field-strength), as is done in conformal supergravity.

2. Gauged orthosymplectic symmetry

The group OSp(1, 4) is the set of linear transformations in the $4\oplus 1$ space

$$\begin{pmatrix}\chi_\alpha\\ \phi\end{pmatrix},$$

with infinitesimal transformations taking the form

$$\delta\begin{pmatrix}\chi_\alpha\\ \phi\end{pmatrix}=\omega\begin{pmatrix}\chi_\alpha\\ \phi\end{pmatrix}, \tag{1}$$

where ω can be parametrised by the 5×5 matrix

$$\omega=\begin{pmatrix}\frac{1}{2}\mathrm{i}(\omega_a\gamma_a+\frac{1}{2}\omega_{ab}\sigma_{ab}) & \epsilon_\alpha\\ \bar{\epsilon}^\alpha & 0\end{pmatrix}. \tag{2}$$

ϵ is a Majorana spinor with $\epsilon=C\bar{\epsilon}^{\mathrm{T}}$ and, provided it anticommutes with χ, the real form

$$\chi^{\mathrm{T}}C^{-1}\chi+\phi^2=\phi^2-\bar{\chi}\chi$$

is left-invariant under the group action.

The fourteen gauge fields are assigned to the antisymmetric rank-two tensor representation and can be written as the 5×5 matrix

$$\Phi_\mu=\begin{pmatrix}\frac{1}{2}\mathrm{i}W_\mu & \psi_\mu\\ \bar{\psi}_\mu & 0\end{pmatrix}, \tag{3}$$

where ψ_μ is the spin-$\frac{3}{2}$ Rarita–Schwinger gauge field and W_μ is the Sp(4) gauge potential,

$$W_\mu=\kappa^{-1}L_\mu{}^a\gamma_a+\tfrac{1}{2}B_\mu{}^{[ab]}\sigma_{ab}.$$

Here $L_\mu{}^a$ and $B_\mu{}^{[ab]}$ are the vierbein and the SL(2, $\mathbb{C}$) connection respectively. In analogy with Yang–Mills theory, the OSp(1, 4) field strengths are given by

$$\Phi_{\mu\nu}=\partial_\mu\Phi_\nu-\partial_\nu\Phi_\mu+[\Phi_\mu,\Phi_\nu]=\begin{pmatrix}\frac{1}{2}\mathrm{i}W_{\mu\nu}+\psi_\mu\bar{\psi}_\nu-\psi_\nu\bar{\psi}_\mu & \psi_{\mu\nu}\\ \bar{\psi}_{\mu\nu} & 0\end{pmatrix} \tag{4}$$

where

$$W_{\mu\nu}=\partial_\mu W_\nu-\partial_\nu W_\mu+\tfrac{1}{2}\mathrm{i}[W_\mu,W_\nu]=B_{\mu\nu}+\tfrac{1}{2}\mathrm{i}[L_\mu,L_\nu]+\nabla_\mu L_\nu-\nabla_\nu L_\mu,$$

$$\psi_{\mu\nu}=\nabla_\mu\psi_\nu-\nabla_\nu\psi_\mu+\tfrac{1}{2}\mathrm{i}(L_\mu\psi_\nu-L_\nu\psi_\mu),$$

$$B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu+\tfrac{1}{2}\mathrm{i}[B_\mu,B_\nu],$$

$$\nabla_\mu L_\nu=\partial_\mu L_\nu+\tfrac{1}{2}\mathrm{i}[B_\mu,L_\nu],\qquad \nabla_\mu\psi_\nu=\partial_\mu\psi_\nu+\tfrac{1}{2}\mathrm{i}[B_\mu,\psi_\nu].$$

Exponentiating the gauge transformations as $\Omega(x)=\exp\omega$,

$$\Phi_{\mu\nu}\to\Omega(x)\Phi_{\mu\nu}\Omega^{-1}(x) \tag{5}$$

under OSp(1, 4), and transforms as an antisymmetric covariant tensor under general coordinate transformations.

A covariant derivative of the field strength with respect to OSp(1, 4) may be defined by

$$\nabla_\rho \Phi_{\mu\nu} = \partial_\rho \Phi_{\mu\nu} + [\Phi_\rho, \Phi_{\mu\nu}]$$
$$= \begin{pmatrix} \tfrac{1}{2}\mathrm{i}(\partial_\rho W_{\mu\nu} + \tfrac{1}{2}\mathrm{i}[W_\rho, W_{\mu\nu}]) + \psi_\rho \bar{\psi}_{\mu\nu} - \psi_{\mu\nu}\bar{\psi}_\rho & \partial_\rho + \tfrac{1}{2}\mathrm{i}\, W_\rho \psi_{\mu\nu} \\ + \partial_\rho(\psi_\mu \bar{\psi}_\nu - \psi_\nu \bar{\psi}_\mu) + \tfrac{1}{2}\mathrm{i}[W_\rho, \psi_\mu\bar{\psi}_\nu - \psi_\nu\bar{\psi}_\mu], & -(\tfrac{1}{2}\mathrm{i}\, W_{\mu\nu} + \psi_\mu\psi_\nu - \psi_\nu\psi_\mu)\psi_\rho \\ \partial_\rho \bar{\psi}_{\mu\nu} - \tfrac{1}{2}\mathrm{i}\bar{\psi}_{\mu\nu} W_\rho & \\ + \bar{\psi}_\rho(\tfrac{1}{2}\mathrm{i}\, W_{\mu\nu} + \psi_\mu\bar{\psi}_\nu - \psi_\nu\bar{\psi}_\mu) & 0 \end{pmatrix}. \tag{6}$$

Since the orthosymplectic symmetry is necessarily broken spontaneously (Chamseddine 1978, Cremmer *et al* 1978), Higgs-like fields must be introduced, and these are taken to transform under the ten-dimensional symmetric second-rank tensor representation of the group. Represented as a 5×5 matrix,

$$C = \begin{pmatrix} H & \lambda_\alpha \\ -\bar{\lambda}^\alpha & \pi(x) \end{pmatrix} \tag{7}$$

where

$$H = \tfrac{1}{4}\pi(x) + \varphi(x)\gamma_5 + V_a(x)\mathrm{i}\gamma_a\gamma_5,$$

$\pi(x)$ is a pseudoscalar, $\varphi(x)$ a scalar, V_a a vector and λ_α a Majorana spinor. C transforms as

$$C \to \Omega(x) C \Omega^{-1}(x) \tag{8}$$

and is a world scalar. Its covariant derivative is a covariant vector

$$\nabla_\mu C = \partial_\mu C + [\Phi_\mu, C]$$
$$= \begin{pmatrix} \nabla_\mu H + \tfrac{1}{2}\mathrm{i}[L_\mu, H] - \psi_\mu\bar{\lambda} - \lambda\bar{\psi}_\mu, & \nabla_\mu\lambda + \tfrac{1}{2}\mathrm{i}L_\mu\lambda + \psi_\mu\pi - H\psi_\mu \\ -\nabla_\mu\bar{\lambda} + \tfrac{1}{2}\mathrm{i}\bar{\lambda}L_\mu - \pi\bar{\psi}_\mu + \bar{\psi}_\mu H, & \partial_\mu\pi + \bar{\psi}_\mu\lambda + \bar{\lambda}\psi_\mu \end{pmatrix}, \tag{9}$$

where

$$\nabla_\mu H = \partial_\mu H + \tfrac{1}{2}\mathrm{i}[B_\mu, H], \qquad \nabla_\mu\lambda = \partial_\mu\lambda + \tfrac{1}{2}\mathrm{i}B_\mu\lambda,$$

and has the transformation

$$\nabla_\mu C \to \Omega(x)\nabla_\mu C \Omega^{-1}(x).$$

The metric tensor is defined† to be the OSp(1, 4) invariant

$$g_{\mu\nu} \sim \mathrm{Tr}(\nabla_\mu C \nabla_\nu C), \tag{10}$$

where $g_{\mu\nu}$ undergoes the usual general coordinate transformation

$$g'_{\mu\nu}(\bar{x}) = \frac{\partial x^\rho}{\partial \bar{x}^\mu}\frac{\partial x^\sigma}{\partial \bar{x}^\nu} g_{\rho\sigma}(x) \tag{11}$$

(which, in analogy with general relativity, allows one to interpret it as the metric tensor).

† A similar metric has been used by Baaklini (1977).

In terms of component fields, one can write the supersymmetric OSp(1, 4) gauge transformations as†

$$\delta L_{\mu a} = -\partial_\mu \omega_a - B_{\mu ba}\omega_b - L_{\mu c}\omega_{ac} - \mathrm{i}\bar{\epsilon}\gamma_a\psi_\mu,$$
$$\delta B_{\mu ab} = -\partial_\mu \omega_{ab} - (L_{\mu a}\omega_b - L_{\mu b}\omega_a) - (B_{\mu cb}\omega_{ac} - B_{\mu ca}\omega_{bc}) - \mathrm{i}\bar{\epsilon}\sigma_{ab}\psi_\mu,$$
$$\delta\psi_\mu = -\partial_\mu\epsilon - \tfrac{1}{2}\mathrm{i}(\omega_a\gamma_a + \tfrac{1}{2}\omega_{ab}\sigma_{ab})\psi_\mu - \tfrac{1}{2}\mathrm{i}(L_{\mu a}\gamma_a + \tfrac{1}{2}B_{\mu ab}\sigma_{ab})\epsilon,$$
$$\delta\pi = 2\bar{\epsilon}\lambda, \qquad \delta\varphi = -\omega_a V_a - \tfrac{1}{2}\bar{\epsilon}\gamma_5\lambda,$$
$$\delta V_a = \omega_a\varphi - \omega_{ab}V_b - \tfrac{1}{2}\bar{\epsilon}\mathrm{i}\gamma_a\gamma_5\lambda,$$
$$\delta\lambda = \tfrac{1}{2}\mathrm{i}(\omega_a\gamma_a + \tfrac{1}{2}\omega_{ab}\sigma_{ab})\lambda - (-\tfrac{3}{4}\pi + \varphi\gamma_5 + V_a\mathrm{i}\gamma_a\gamma_5)\epsilon. \tag{12}$$

3. Spontaneous symmetry breaking and the Lagrangian

In order for the theory to possess a stable, flat (Poincaré invariant) vacuum $\langle L_{\mu a}\rangle = \eta_{\mu a}$, one requires the multiplet C to have a non-vanishing expectation value. This may be implemented‡ by imposing two group-invariant constraint equations§ which reduce the number of independent components of C from ten to eight, which is the correct number to parametrise the homogeneous space OSp(1, 4)/SL(2, ℂ). Further, as mentioned above, using the gauge freedom afforded by the parameters ω_a, ϵ_α, one eliminates the fields V_a and λ‖ so that the multiplet C assumes the form

$$C = \langle C\rangle = \begin{pmatrix} \tilde{a}\gamma_5 & 0 \\ 0 & 0 \end{pmatrix}, \tag{13}$$

where $\tilde{a}$ is some constant. This then allows $\nabla_\mu C$ to be written in the simple form

$$\nabla_\mu C = \tilde{a}\begin{pmatrix} L_\mu{}^a \mathrm{i}\gamma_a\gamma_5 & -\gamma_5\psi_\mu \\ \bar{\psi}_\mu\gamma_5 & 0 \end{pmatrix}. \tag{14}$$

The metric tensor reduces to

$$g_{\rho\lambda} \equiv -\tfrac{1}{4}\tilde{a}^{-2}\,\mathrm{Tr}(\nabla_\rho C \nabla_\lambda C) = L_\rho{}^a L_\lambda{}^a + \tfrac{1}{2}\bar{\psi}_\rho\psi_\lambda, \tag{15}$$

by which relation it is defined in terms of the dynamical fields $L_\mu{}^a$ and ψ_μ.

Before constructing a Lagrangian, the question of general coordinate invariance requires some discussion. As stated previously, one does not have to insist upon the coordinate invariance of the action but, in keeping with the spirit of general relativity, it certainly seems desirable. To this end, the covariant derivative, equations (6) and (9),

† From which it may be surmised that V_a, λ may be set to zero by a suitable choice of the parameters ϵ_α and ω_a. We shall in fact work in a gauge specified by this particular choice (designated the unitary gauge). After symmetry breaking, the residual SL(2, ℂ) gauge freedom is eliminated by using the Lie parameter ω_{ab} to make $L_{\mu a}$ symmetric.

‡ Alternatively, one may proceed along more conventional lines by minimising the Higgs potential (see Chamseddine 1977, 1978) to obtain the desired symmetry breaking.

§ The group-invariant constraints, which Chamseddine (1978) takes to be Tr $C^2 = -4\tilde{a}^2$ and Tr $C^3 = 0$, have a dynamical content with which we are not concerned here except to the extent that they allow $\pi(x)$ and $\varphi(x)$ to be eliminated in favour of $V_a(x)$ and λ_α, which play the role of the 'preferred' fields of non-linear realisations (Chamseddine *et al* 1978, Abdus Salam and Strathdee 1969, Zumino 1977).

‖ Following Chamseddine (1977), one may show that the effect of the two constraint equations is to cause $\langle\delta V_a\rangle$, $\langle\delta\lambda_\alpha\rangle$ to acquire C-number displacements, which indicates the non-invariance of the vacuum and the role of V_a and λ_α as Goldstone fields.

will have to be generalised as follows to take into account both the OSp(1, 4) 'internal symmetry' and general covariance. After the manner of a Cartan covariant derivative, we define

$$\mathscr{D}_\rho \Phi_{\mu\nu} = \nabla_\rho \Phi_{\mu\nu} - \Gamma_\rho{}^\kappa{}_\mu \Phi_{\kappa\nu} - \Gamma_\rho{}^\kappa{}_\nu \Phi_{\mu\kappa}$$
$$= \partial_\rho \Phi_{\mu\nu} + [\Phi_\rho, \Phi_{\mu\nu}] - \Gamma_\rho{}^\kappa{}_\mu \Phi_{\kappa\nu} - \Gamma_\rho{}^\kappa{}_\nu \Phi_{\mu\kappa} \tag{16}$$

and

$$\mathscr{D}_\nu(\nabla_\mu C) = \partial_\nu(\nabla_\mu C) + [\Phi_\nu, \nabla_\mu C] - \Gamma_\nu{}^\lambda{}_\mu(\nabla_\lambda C). \tag{17}$$

The generalised connection $\Gamma^\kappa_{\rho\mu}$ is determined in terms of the dynamical fields by the condition that the covariant derivative of the metric tensor vanishes†:

$$\mathscr{D}_\mu g_{\rho\lambda} = \mathscr{D}_\mu \operatorname{Tr}(\nabla_\rho C \nabla_\lambda C) = 0, \tag{18}$$

which gives the following expression for the symmetric connection after inversion in the standard manner (the contragradient metric tensor $g^{\mu\nu}$ is defined by $g^{\mu\nu}g_{\nu\lambda} = \delta^\mu{}_\lambda$):

$$\Gamma^\kappa_{\mu\lambda} = \tfrac{1}{2} g^{\rho\kappa}[(\partial_\mu L_\rho{}^a - \partial_\rho L_\mu{}^a)L_\lambda{}^a + (\partial_\lambda L_\rho{}^a - \partial_\rho L_\lambda{}^a)L_\mu{}^a + (\partial_\mu L_\lambda{}^a + \partial_\lambda L_\mu{}^a)L_\rho{}^a$$
$$- (\tilde{X}_{\mu\rho\lambda} + \tilde{X}_{\lambda\rho\mu}) - (\tilde{X}_{\mu\lambda\rho} + \tilde{X}_{\lambda\mu\rho}) + (\tilde{X}_{\rho\mu\lambda} + \tilde{X}_{\rho\lambda\mu})], \tag{19}$$

where the Majorana spinor terms, $\tilde{X}$, are given by

$$\tilde{X}_{\mu\rho\lambda} = -\tfrac{1}{4}[\bar{\psi}_\lambda(\partial_\mu \psi_\rho) + (\partial_\mu \bar{\psi}_\rho)\psi_\lambda + \mathrm{i} L_\mu{}^a \bar{\psi}_\rho \gamma_a \psi_\lambda + \tfrac{1}{2}\mathrm{i} B_{\mu ab} \bar{\psi}_\lambda \sigma_{ab} \psi_\rho]. \tag{20}$$

One is now in a position to write down OSp(1, 4) and general coordinate invariant Lagrangians constructed from the metric tensor, the 'Higgs' multiplet C and the covariant derivatives of the field strengths. However, the crucial point is that, to secure renormalisability, the propagators of the theory should have a high-energy behaviour of at least k^{-4} (in analogy with ordinary R^2 gravity (Stelle 1977)). This of course means that at the linearised level $h_{\mu a}$ (where $L_{\mu a} = \eta_{\mu a} + \kappa h_{\mu a}$)‡ should have kinetic terms containing minimally four derivatives. It turns out that this restriction eliminates candidates such as (*a*) $g^{\rho\sigma}\epsilon^{\mu\nu\kappa\lambda} \operatorname{Tr}[(\nabla_\rho C)(D_\sigma \Phi_{\mu\nu})\Phi_{\kappa\lambda}]$ and (*b*) $g^{\rho\sigma}\epsilon^{\mu\nu\kappa\lambda} \operatorname{Tr}[C(D_\rho \Phi_{\mu\nu})(D_\sigma \Phi_{\kappa\lambda})]$. This is clear from what follows. In case (*a*), working in the unitary gauge in which $\nabla_\rho C = \mathrm{i} L_\rho \gamma_5$, partial integrations allow the Lagrangian to be written as $g^{\rho\sigma}\epsilon^{\mu\nu\kappa\lambda} \operatorname{Tr}[(D_\sigma \gamma_5 L_\rho)\Phi_{\mu\nu}\Phi_{\kappa\lambda}]$. In this form, it can easily be seen that the γ structure causes all higher-derivative terms to vanish. The Lagrangian (*b*), although containing higher-derivative terms (which lead to a propagator high-energy behaviour of $\langle T(hh)\rangle \sim k^{-4}$, $\langle T(BB)\rangle \sim k^{-4}$ and $\langle T(Bh)\rangle \sim k^{-3}$), is not power-counting renormalisable in the sense that not all one-loop divergences can be absorbed into the original Lagrangian. One is thus led to a unique choice for the higher-derivative-containing piece of the Lagrangian.

The proposed Lagrangian is

$$\mathscr{L} = (g_1/\tilde{a})\,\epsilon^{\mu\nu\kappa\lambda} \operatorname{Tr}(C\Phi_{\mu\nu}\Phi_{\kappa\lambda}) + g_2\sqrt{-g}\, g^{\rho\sigma} g^{\kappa\mu} g^{\lambda\nu} \operatorname{Tr}(\mathscr{D}_\sigma \Phi_{\kappa\lambda} \mathscr{D}_\rho \Phi_{\mu\nu})$$
$$+ (g_3/4\tilde{a}^5)\epsilon^{\mu\nu\kappa\lambda} \operatorname{Tr}(C\nabla_\mu C \nabla_\nu C \nabla_\kappa C \nabla_\lambda C)$$
$$+ (g_4/2\tilde{a}^3)\epsilon^{\mu\nu\kappa\lambda} \operatorname{Tr}(C\nabla_\mu C \nabla_\nu C \Phi_{\kappa\lambda}) + \alpha \operatorname{Tr}(\partial_\lambda \partial_\mu \Phi_\mu \partial_\lambda \partial_\nu \Phi_\nu). \tag{21}$$

† Since the metric tensor is symmetric, equation (18) gives forty constraints and one may in addition demand that the connections $\Gamma^\kappa_{\rho\lambda}$ so derived are symmetric as well, so that they give the same number of relations. Thus one has in fact a spin-containing connection which is torsionless (see footnote 2 of Kaku *et al* 1977).

‡ The symmetry breaking of OSp(1, 4) implied by $\langle L_{\mu a}\rangle = \eta_{\mu a}$ allows the identification of Greek and Latin indices, providing the link between Poincaré transformations and the SL(2, $\mathbb{C}$) index transformations. One may then interpret $h_{\mu a}$ as being the graviton field.

The first term (with dimensional coupling g_1) contains (see Chamseddine 1977, 1978) the usual Einstein supergravity action (with a cosmological term) coupled minimally to the spin-$\frac{3}{2}$ field, with a mass-like term appearing for the Rarita–Schwinger field.

There are four arbitrary dimensional coupling constants, g_1, g_2, g_3 and g_4, in the theory to begin with. The constraints, that the Einstein part comes in with the correct factor (in terms of the Einstein constant κ), and the vanishing of the cosmological term†, reduce the number of independent parameters to two. A further constraint is provided if one wants to adjust the mass of the spin-$\frac{3}{2}$ gauge field independently (otherwise it is fixed at the order of the Planck mass). This leaves one independent coupling constant‡.

The gauge fixing term $\alpha\ \mathrm{Tr}\ \partial_\lambda\partial_\mu\Phi_\mu\partial_\lambda\partial_\nu\Phi_\nu$ is included as usual to break the general coordinate invariance of the theory (the particular form chosen is for simplicity of the momentum-space propagators).

4. The linearised theory

We next examine the content of the theory at the linearised level. Since even the bilinear contribution is fairly complicated, we shall only consider the bosonic sector in the unitary gauge. Treating all fields (except $L_{\mu a}$) $h_{\mu a}$, $B_{\mu ab}$, ψ_μ as perturbations, the bilinear terms are contained in the general expression

$$\mathscr{L}_{\substack{\text{Bosonic,}\\ \text{Bilinear}}} = \mathscr{L}_{\mathrm{I}} + \mathscr{L}_{\mathrm{II}} + \mathscr{L}_{\mathrm{III}}, \tag{22a}$$

where

$$\begin{aligned}\mathscr{L}_{\mathrm{I}} = -2g_2'\kappa^2(&\partial^2 h_{\mu\nu}\partial^2 h_{\mu\nu} - \partial_\mu\partial_\lambda h_{\lambda\kappa}\partial^2 h_{\kappa\mu} + a\partial_\mu h_{\nu\nu}\partial_\mu h_{\lambda\lambda} + b\partial_\mu h_{\mu\nu}\partial_\nu h_{\lambda\lambda}\\ &+ c\partial_\mu h_{\nu\lambda}\partial_\mu h_{\nu\lambda} + d\partial_\mu h_{\nu\lambda}\partial_\lambda h_{\mu\nu} + e h_{\mu\mu}h_{\nu\nu} + f h_{\mu\nu}h_{\mu\nu})\\ &- \alpha'\kappa^2\partial_\mu\partial_\nu h_{\mu\kappa}\partial^2 h_{\nu\kappa} + \kappa h_{\mu\nu}T_{\mu\nu},\end{aligned} \tag{22b}$$

$$\begin{aligned}\mathscr{L}_{\mathrm{II}} = -2g_2'\kappa^2(&a_1 B_{\mu\mu\nu}\partial_\lambda h_{\nu\lambda} + b_1 B_{\mu\mu\nu}\partial_\nu h_{\lambda\lambda} + c_1 B_{\mu\nu\lambda}\partial_\lambda h_{\mu\nu} + d_1\partial^2 h_{\nu\lambda}\partial_\kappa B_{\lambda\nu\kappa}\\ &+ e_1\partial_\mu\partial_\nu h_{\lambda\mu}\partial_\kappa B_{\kappa\lambda\nu} + f_1\partial_\mu\partial_\nu h_{\mu\nu}\partial_\lambda B_{\kappa\kappa\lambda} + a_2\partial_\mu\partial_\nu h_{\lambda\mu}\partial_\nu B_{\kappa\kappa\lambda}),\end{aligned} \tag{22c}$$

and

$$\begin{aligned}\mathscr{L}_{\mathrm{III}} = -2g_2'\kappa^2(&\tfrac{1}{2}\partial^2 B_{\mu\nu\lambda}\partial^2 B_{\mu\nu\lambda} - \tfrac{1}{2}\partial_\mu\partial_\lambda B_{\lambda\kappa\nu}\partial^2 B_{\mu\kappa\nu} + a_3\partial_\mu B_{\nu\lambda\mu}\partial_\kappa B_{\nu\lambda\kappa}\\ &+ b_3\partial_\mu B_{\mu\nu\lambda}\partial_\kappa B_{\nu\lambda\kappa} + c_3\partial_\mu B_{\nu\mu\lambda}\partial_\nu B_{\kappa\kappa\lambda} + d_3\partial_\mu B_{\nu\lambda\kappa}\partial_\mu B_{\nu\lambda\kappa}\\ &+ e_3\partial_\mu B_{\nu\lambda\kappa}\partial_\mu B_{\lambda\kappa\nu} + f_3\partial_\mu B_{\mu\nu\lambda}\partial_\kappa B_{\kappa\nu\lambda}\\ &+ a_4 B_{\kappa\kappa\mu}B_{\lambda\lambda\mu} + b_4 B_{\mu\nu\lambda}B_{\nu\mu\lambda} + c_4 B_{\mu\nu\lambda}B_{\mu\nu\lambda})\\ &- \tfrac{1}{2}\alpha'\ \kappa^2\partial_\mu\partial_\nu B_{\mu\kappa\lambda}\partial^2 B_{\nu\kappa\lambda} + \kappa B_{\mu\nu\lambda}J_{\mu\nu\lambda},\end{aligned} \tag{22d}$$

where the coefficients a, b, $c, \ldots, c_4$ are in general dimensional. $T_{\mu\nu}$ and $J_{\mu\nu\lambda}$ are sources coupled to $h_{\mu\nu}$ and $B_{\mu\nu\lambda}$ respectively.

† The alternative is that one would have to quantise in a de Sitter universe if an effective cosmological term were allowed to remain after symmetry breaking.

‡ This is to be contrasted with ordinary R^2 gravity (Stellé 1977), where the action $\sqrt{-g}\ (\kappa^{-2}R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu})$ has two independent (although not perhaps in the renormalisation-group sense, see Abdus Salam and Strathdee (1978)) couplings. The remaining free parameter in the present theory will presumably be necessary to adjust the higher-derivative induced corrections to the $1/r$ Newtonian potential in the classical limit (see Stelle 1977).

Contact with the Lagrangian equation (21) of § 3 is made by the following identifications:

$g_2 = g_2' \kappa^2, \qquad g_1 = g_1' \kappa^2, \qquad g_3 = g_3' \kappa^2, \qquad g_4 = g_4' \kappa^2, \qquad \alpha = \alpha' \kappa^2,$

(where g_1' to g_4' and α' are dimensionless)

$a = -2/\kappa^2, \qquad b = 4/\kappa^2, \qquad c = 11/\kappa^2, \qquad d = -13/\kappa^2,$

$e = -f = (6/\kappa^4)[1 - g_1'/g_2' - (g_3' + g_4')/g_2'], \qquad a_1 = -b_1 = 2(2g_1' + g_4')/g_2'\kappa^3,$

$c_1 = (2/\kappa^3)(9 + 2g_1'/g_2' + g_4'/g_2'), \qquad d_1 = -e_1 = 6/\kappa, \qquad f_1 = -a_2 = 2/\kappa,$

$a_3 = 4/\kappa^2, \qquad b_3 = 6/\kappa^2, \qquad c_3 = d_3 = 2/\kappa^2, \qquad e_3 = -f_3 = 1/\kappa^2,$

$a_4 = -(2g_1' + g_4')/g_2'\kappa^4, \qquad b_4 = -9/\kappa^4 + 2g_1'/g_2'\kappa^4 + g_4'/g_2'\kappa^4, \qquad c_4 = 9/\kappa^4.$

At this linearised level, cancelling the cosmological term would mean setting $e = f = 0$, i.e. taking $g_2' = g_1' + g_3' + g_4'$.

To obtain an insight into the linearised higher-derivative Lagrangian, it may be written symbolically as

$$\mathscr{L}_{\substack{\text{Bosonic}\\ \text{Bilinear}}} = g_2(h\partial^4 h + \kappa^{-2} h\partial^2 h + \kappa^{-4} hh + \kappa^{-3} B\partial h + \kappa^{-1} B\partial^3 h + \kappa^{-2} B\partial^2 B + B\partial^4 B + \kappa^{-4} BB). \tag{23}$$

The salient new features are that the SL(2, $\mathbb{C}$) connection gauge field $B_{\mu ab}$ now propagates and is coupled to the graviton field $h_{\mu\nu}$ by non-trivial mixing terms. From the point of view of renormalisability, the propagator high-energy behaviour is (again symbolically)† as follows:

$$\langle T(hh)\rangle \sim k^{-4}, \qquad \langle T(BB)\rangle \sim k^{-4}, \qquad \langle T(hB)\rangle \sim k^{-5}. \tag{24}$$

An examination of the various one-particle irreducible divergent structures reveals this momentum-space behaviour to be sufficient to ensure renormalisability of the theory (at the one-loop level at least).

The field $h_{\mu\nu}$ describes the usual massless spin-2 graviton, and in addition will have a massive spin-2 excitation (a metric ghost state) and a positive energy massive scalar (Stelle 1977). The twenty-four component (massive) field $B_{\mu[ab]}$ has the spin decomposition‡

$$B_{\mu[ab]} = 2^+ \oplus 1^+ \oplus 1^- \oplus 0^+ \oplus 2^- \oplus 1^- \oplus 1^+ \oplus 0^-.$$

5. Conclusions

The OSp(1, 4) supersymmetric extension of the R^2, $R_{\mu\nu}R^{\mu\nu}$ fourth-order derivative action of ordinary gravity has been presented here. Central to this formulation is the idea of spontaneous symmetry-breaking implemented by group-invariant constraints which simplify the situation in that they allow the dynamical role of two of the

† The exact form of the propagators has been obtained by a lengthy computer calculation.

‡ The particle spectrum of this theory may be compared with the bosonic sector spectrum of Ferrara *et al* (1978). However, the propagating SL(2, $\mathbb{C}$) gauge field $B_{\mu ab}$ in the present theory evidently introduces further spin-2, spin-1 and scalar components.

component fields to be suppressed. As has been shown by Chamseddine (1978) for the case of the Einstein part of the Lagrangian (21), equivalence with the conventional supergravity action can be achieved via a Wigner–Inönu group contraction. However, in the present context the propagating field $B_{\mu ab}$ seems to obscure the relation between the present theory and its group-contracted Poincaré version.

We close with some remarks about the negative norm states (ghosts) in the theory. Obviously any physical interpretation of such a theory is not possible unless the ghosts can be decoupled from the physical sector, in the sense of Abdus Salam and Strathdee (1978) and Julve and Tonin (1978) for instance. However, as stated by Babelon and Namazie (1978), in the case of supersymmetric theories, relations between the renormalisation group functions may prove to be an obstacle to the Salam–Strathdee consistency condition for the elimination of ghosts being fulfilled.

Acknowledgments

The author is indebted to Professors Abdus Salam and J Strathdee for suggesting the problem, for discussions and encouragement during the course of the work, and also to Dr N S Baaklini for helpful conversations and for a critical reading of the manuscript. Thanks are due to Drs O Babelon and D Storey for several useful remarks. He wishes to thank Professor Abdus Salam, the International Atomic Energy Agency and Unesco for hospitality at the International Centre for Theoretical Physics, Trieste.

Appendix 1

In this appendix we outline the expansion of the Lagrangian equation (21) into its bilinear and higher-order components of its purely bosonic sector. The calculation is extremely lengthy and was in part performed by computer†. The closure of the Dirac γ algebra enables one to write

$$\mathscr{D}_\sigma W_{\mu\nu} \equiv \tfrac{1}{2}\mathrm{i}(W_{\mu\nu;\sigma}^{\ \ a}\gamma_a + W_{\mu\nu;\sigma}^{\ \ ab}\sigma_{ab}) \tag{A1.1}$$

and

$$W_{\mu\nu} \equiv W_{\mu\nu}^{\ \ a}\gamma_a + W_{\mu\nu}^{\ \ ab}\sigma_{ab}, \tag{A1.2}$$

where

$$W_{\mu\nu;\sigma}^{\ \ a} = \partial_\sigma W_{\mu\nu}^{\ \ a} + 2L_\sigma^{\ c}W_{\mu\nu}^{\ \ ac} - B_\sigma^{\ ac}W_{\mu\nu}^{\ \ c} - \Gamma_{\sigma\mu}^{\ \ \alpha}W_{\alpha\nu}^{\ \ a} - \Gamma_{\sigma\nu}^{\ \ \alpha}W_{\mu\alpha}^{\ \ a}, \tag{A1.3}$$

$$W_{\mu\nu;\sigma}^{\ \ ab} = \partial_\sigma W_{\mu\nu}^{\ \ ab} + L_\sigma^{\ [a}W_{\mu\nu}^{\ \ b]} + B_\sigma^{\ ac}W_{\mu\nu}^{\ \ bc} - B_\sigma^{\ bc}W_{\mu\nu}^{\ \ ac} - \Gamma_{\sigma\mu}^{\ \ \alpha}W_{\alpha\nu}^{\ \ ab} - \Gamma_{\sigma\nu}^{\ \ \alpha}W_{\mu\alpha}^{\ \ ab}, \tag{A1.4}$$

$$W_{\mu\nu}^{\ \ ab} = B_{\mu\nu}^{\ \ ab} + L_{[\mu}^{\ \ a}L_{\nu]}^{\ \ b}, \tag{A1.5}$$

$$W_{\mu\nu}^{\ \ a} = L_{\nu;\mu}^{\ \ a} - L_{\mu;\nu}^{\ \ a}, \tag{A1.6}$$

$$B_{\mu\nu}^{\ \ ab} = \tfrac{1}{2}(\partial_\mu B_\nu^{\ ab} - \partial_\nu B_\mu^{\ ab}) + B_{[\mu}^{\ \ ac}B_{\nu]}^{\ \ bc}, \tag{A1.7}$$

$$L_{\nu;\ \mu}^{\ \ a} = \partial_\mu L_\nu^{\ a} - B_\mu^{\ ac}L_\nu^{\ c}, \tag{A1.8}$$

with $A_{[a}B_{b]} = \tfrac{1}{2}(A_aB_b - A_bB_a)$; the Einstein constant κ has been set to unity.

† The symbolic manipulation programme 'SCHOONSCHIP' (Veltman 1967) was used; see also Strubbe (1974).

Also needed are the symmetric connection defined by equation (19) and the product $\sqrt{-g}\, g^{\rho\sigma}g^{\kappa\mu}g^{\lambda\nu}$, both to second order in the graviton field $h_{\mu\nu}$. They are given by the expressions

$$\Gamma_{\mu\lambda}^{\ \ \kappa} = \underset{-}{\Gamma}_{\mu\lambda}^{\ \ \kappa} + \underset{=}{\Gamma}_{\mu\lambda}^{\ \ \kappa} + \dots, \tag{A1.9}$$

where

$$\underset{-}{\Gamma}_{\mu\lambda}^{\ \ \kappa} = \eta^{\kappa\rho}(\partial_\mu h_{\rho\lambda} - \partial_\rho h_{\mu\lambda} + \partial_\lambda h_{\mu\rho}), \tag{A1.10}$$

$$\underset{=}{\Gamma}_{\mu\lambda}^{\ \ \kappa} = \tfrac{1}{2}[h_\lambda^{\ \rho}(\partial_\mu h_{\kappa\rho} - \partial_\kappa h_{\mu\rho}) + h_\kappa^{\ \rho}(\partial_\mu h_{\lambda\rho} + \partial_\lambda h_{\mu\rho}) + h_\mu^{\ \rho}(\partial_\lambda h_{\kappa\rho} - \partial_\kappa h_{\lambda\rho})]$$
$$+ 2h^{\kappa\rho}(\partial_\mu h_{\rho\lambda} - \partial_\rho h_{\mu\lambda} + \partial_\lambda h_{\mu\rho}), \tag{A1.11}$$

and

$$\sqrt{-g}\, g^{\rho\sigma}g^{\kappa\mu}g^{\lambda\nu} = H_{\rho\sigma\kappa\mu\lambda\nu} + \underset{-}{H}_{\rho\sigma\kappa\mu\lambda\nu} + \underset{=}{H}_{\rho\sigma\kappa\mu\lambda\nu} + \dots \tag{A1.12}$$

where†

$$H_{\rho\sigma\kappa\mu\lambda\nu} = \eta_{\rho\sigma}\eta_{\kappa\mu}\eta_{\lambda\nu}, \tag{A1.13}$$

$$\underset{-}{H}_{\rho\sigma\kappa\mu\lambda\nu} = \eta_{\rho\sigma}(h_{\alpha\alpha}\eta_{\kappa\mu}\eta_{\lambda\nu} - 2h_{\kappa\mu}\eta_{\lambda\nu} - 2h_{\lambda\nu}\eta_{\kappa\mu}) - 2h_{\rho\sigma}\eta_{\kappa\mu}\eta_{\lambda\nu}, \tag{A1.14}$$

$$\underset{=}{H}_{\rho\sigma\kappa\mu\lambda\nu} = \tfrac{1}{2}(h_{\alpha\alpha}h_{\beta\beta} - h_{\alpha\beta}h_{\alpha\beta})\eta_{\rho\sigma}\eta_{\kappa\mu}\eta_{\lambda\nu} + 4(h_{\rho\sigma}h_{\kappa\mu}\eta_{\lambda\nu} + h_{\rho\sigma}h_{\lambda\nu}\eta_{\kappa\mu} + h_{\kappa\mu}h_{\lambda\nu}\eta_{\rho\sigma})$$
$$-2h_{\alpha\alpha}(h_{\rho\sigma}\eta_{\kappa\mu}\eta_{\lambda\nu} + h_{\kappa\mu}\eta_{\rho\sigma}\eta_{\lambda\nu} + h_{\lambda\nu}\eta_{\rho\sigma}\eta_{\kappa\mu})$$
$$+3(h_{\rho\alpha}h_{\sigma\alpha}\eta_{\kappa\mu}\eta_{\lambda\nu} + h_{\kappa\alpha}h_{\mu\alpha}\eta_{\rho\sigma}\eta_{\nu\lambda} + h_{\lambda\alpha}h_{\nu\alpha}\eta_{\rho\sigma}\eta_{\mu\kappa}). \tag{A1.15}$$

The Einstein part of equation (21) is then given by

$$\mathscr{L}_{\substack{\text{Einstein,}\\ \text{Bosonic, Bilinear}}} = -g_1\epsilon^{\mu\nu\kappa\lambda}\epsilon_{abcd}W_{\mu\nu}^{\ \ ab}W_{\kappa\lambda}^{\ \ cd} \tag{A1.16}$$

and the higher-derivative Lagrangian by‡

$$\mathscr{L}_{\substack{\text{Higher Derivative,}\\ \text{Bosonic, Bilinear}}} = -(H_{\rho\sigma\kappa\mu\lambda\nu} + \underset{-}{H}_{\rho\sigma\kappa\mu\lambda\nu} + \underset{=}{H}_{\rho\sigma\kappa\mu\lambda\nu})$$
$$\times(W_{\kappa\lambda;\sigma}^{\ \ a}W_{\mu\nu;\rho}^{\ \ a} + 2W_{\kappa\lambda;\sigma}^{\ \ ab}W_{\mu\nu;\rho}^{\ \ ab}). \tag{A1.17}$$

Our conventions are $\eta_{\mu\nu} = \mathrm{diag}(+---)$, $\sigma_{ab} = \frac{1}{2}\mathrm{i}[\gamma_a, \gamma_b]$ and $(\gamma_5)^2 = -1$. The density $\epsilon^{\mu\nu\kappa\lambda}$ is defined to be $+1$ if $(\mu\nu\kappa\lambda)$ is an even permutation of (0123), -1 if $(\mu\nu\kappa\lambda)$ is an odd permutation of (0123), and zero otherwise; $\epsilon^{\mu\nu\kappa\lambda}\epsilon_{\mu\nu\kappa\lambda} = -24$. Also $\det(L_{\mu a})\,\epsilon_{\mu\nu\kappa\lambda} = L_{\mu a}L_{\nu b}L_{\kappa c}L_{\lambda d}\epsilon_{abcd}$.

† With $\psi_\mu = 0$, $\sqrt{-g} = \det(L_{\mu a})$ and $g^{\mu\nu}$ expanded to second order in $h^{\mu\nu}$ is given by $g^{\mu\nu} = \eta^{\mu\nu} - 2h^{\mu\nu} + 3h^{\mu a}h^{\nu a} + \dots$.

‡ Since at this stage a non-covariant decomposition has been made, there is no need to distinguish between upper and lower indices.

Appendix 2

The equations of motion of the linearised Lagrangian are appended for completeness. For the Lagrangian $\mathscr{L}_{\substack{\text{Bosonic}\\ \text{Bilinear}}} = \mathscr{L}_{\text{I}}+\mathscr{L}_{\text{II}}+\mathscr{L}_{\text{III}}$, these are given by the two coupled equations

$$M^{\rho\sigma}_{\gamma\delta}h_{\rho\sigma}+\tilde{M}^{\rho\sigma\lambda}_{\gamma\delta}B_{\rho\sigma\lambda}=-\kappa^{-1}T_{(\gamma\delta)}, \tag{A2.1}$$

$$N^{\rho\sigma\lambda}_{\gamma\delta\epsilon}B_{\rho\sigma\lambda}+\tilde{N}^{\rho\sigma}_{\gamma\delta\epsilon}h_{\rho\sigma}=-\kappa^{-1}J_{\gamma[\delta\epsilon]}. \tag{A2.2}$$

M, $\tilde{M}$, N and $\tilde{N}$ are given by the matrices

$$\begin{aligned}(Mh)_{\gamma\delta}=&[(2g_2'-\alpha')\Box+2g_2'd](h_{\delta,\gamma}+h_{\gamma,\delta})-2g_2'(\Box^2-2c\Box+2f)h_{\gamma\delta}\\&+2g_2'(2a\eta_{\gamma\delta}\Box+b\partial_\gamma\partial_\delta-2e\eta_{\gamma\delta})h+2g_2'b\eta_{\gamma\delta}h_{\mu,\mu},\end{aligned} \tag{A2.3}$$

$$\begin{aligned}(\tilde{M}B)_{\gamma\delta}=&-g_2'[(d_1\Box-c_1)(B_{\delta,\gamma}+B_{\gamma,\delta})+(a_2\Box-a_1)(\partial_\delta B_{\mu\mu\gamma}+\partial_\gamma B_{\mu\mu\delta})\\&+e_1(\partial_\delta B_\gamma+\partial_\gamma B_\delta)+(f_1\partial_\gamma\partial_\delta-b_1\eta_{\gamma\delta})B_{\mu,\mu}],\end{aligned} \tag{A2.4}$$

$$\begin{aligned}(NB)_{\gamma\delta\epsilon}=&g_2'[-\Box^2+2d_3\Box-2c_4]B_{\gamma\delta\epsilon}-2g_2'[e_3\Box+b_4]B_{\delta\gamma\epsilon}+(\tfrac{1}{2}c_3\partial_\gamma\partial_\delta-a_4\eta_{\gamma\delta})B_{\kappa\kappa\epsilon}\\&+[(g_2'-\alpha'/2)\Box+2g_2'f_3]\partial_\gamma B_{\delta\epsilon}+g_2'b_3\partial_\epsilon B_{\gamma\delta}\\&+g_2'(2a_3\partial_\epsilon B_{\gamma,\delta}+b_3\partial_\gamma B_{\delta,\epsilon}-c_3\eta_{\gamma\delta}B_\epsilon)-(\delta\leftrightarrow\epsilon),\end{aligned} \tag{A2.5}$$

$$\begin{aligned}(\tilde{N}h)_{\gamma\delta\epsilon}=&g_2'\{d_1\Box\partial_\epsilon h_{\delta\gamma}+[e_1\partial_\gamma\partial_\epsilon-\eta_{\gamma\epsilon}(a_2\Box-a_1)]h_\delta+f_1\eta_{\gamma\delta}\partial_\epsilon h_{\mu,\mu}\\&+b_1\eta_{\gamma\epsilon}\partial_\delta h+c_1\partial_\delta h_{\gamma\epsilon}\}-(\delta\leftrightarrow\epsilon),\end{aligned} \tag{A2.6}$$

where the coefficients $a, \ldots, c_4$ have been defined in § 4, and the following notation is used:

$$\partial_\mu h_{\mu\nu}\equiv h_\nu, \qquad \partial_\epsilon B_{\delta\gamma\epsilon}=B_{\delta,\gamma}, \qquad h_{\mu\mu}\equiv h,$$

$$\partial_\mu B_{\mu\delta\epsilon}=B_{\delta\epsilon}, \qquad \partial_\lambda\partial_\nu h_{\mu\nu}\equiv h_{\mu,\lambda}, \qquad \partial_\mu\partial_\kappa B_{\kappa\gamma\mu}=\partial_\mu\partial_\kappa B_{\mu\gamma\kappa}=B_\gamma,$$

$$T_{(\gamma\delta)}=\tfrac{1}{2}(T_{\gamma\delta}+T_{\delta\gamma}), \qquad J_{\gamma[\delta\epsilon]}=\tfrac{1}{2}(J_{\gamma\delta\epsilon}-J_{\gamma\epsilon\delta}).$$

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